sparse if order $\tau = n^{1+\gamma}$, $\gamma < 1$. For most classes of sparse matrices occurring in connection with electrical problems a value of $\gamma = 0.2$ or smaller is typical [4]. For many important sparse matrices (such as banded matrices), the nonzeroes are evenly distributed among the rows. There are, however, other highly sparse matrices (such as doubly bordered matrices) where the number of nonzeroes in at least one row is precisely $n$. Row sorting thus becomes a process of order $n^2$. This compares quite unfavorably with all other operations with sparse matrices, which tend to be of order $n^{1+2\gamma}$ or less [4]. A possibility for overcoming this limitation is to use some "logarithmic" sort algorithm, such as quicksort or heapsort. The number of comparisons required to sort each row is of order $\eta \log \eta$. This is an improvement for matrices with some dense rows (such as bordered matrices) but in general can be expected to be undesirable for short rows.

It has been determined by the author that a much more efficient algorithm for sorting an arbitrary sparse matrix is a process which can be called a "simultaneous radix sort." This algorithm works by destroying the row linked lists to form column linked lists with row indices and then it destroys the newly formed column linked lists to reform the row linked lists with column indices. If the temporary column linked lists are accessed in reverse order while reforming the row linked lists, then the elements removed from higher numbered column linked lists will be added to the row linked lists before those from lower numbered column lists. This will eventually place elements in the higher numbered columns behind those with lower numbered columns in their respective linked lists. Fig. 1 illustrates a Fortran implementation of the algorithm by two consecutive calls to a subroutine RSORT, the first to form column linked lists and the second to reform the row linked lists. $N$ is the dimension of the matrix, RP is a vector of row head pointers, CP is a *temporary* vector of column head pointers, CI is a vector of column indices (which is temporarily also used for row indices) and NE is a vector of "next element" pointers in the linked list. H1 and H2 are dummy names within RSORT, which correspond to RP and CP or vice versa. An inspection of the algorithm reveals that it is of order $\tau$ and requires no comparisons among indices. After suitable modification, the algorithm works equally well if the ordering is to be performed on an arbitrary permutation of the column indices.

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## Realization of Two-Port Parameters

SUMER CAN

*Abstract*—A universal active-*RC* network is proposed for the realization of two-port parameters. It is shown that by generalizing Brugler's network, it is possible to synthesize arbitrary current transfer functions, transfer impedances, transfer admittances, driving-point admittances, as well as voltage transfer functions, by using simple functional decompositions. Comparing to the similar realizations, the proposed network offers a reduction in the number of active elements.

### Introduction

An active-*RC* network containing two operational amplifiers which can be used for the realization of arbitrary voltage transfer functions has been given by Lovering [1]. Recently, it was shown that a generalization of Lovering's circuit results in an active-*RC* network [2] which can be used to realize an arbitrary function as any of the five two-port parameters: driving-point admittance, transfer admittance, transfer impedance, current transfer function, and voltage transfer function.

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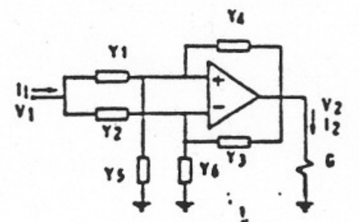

Fig. 1.

Another active-*RC* network containing a single operational amplifier, which can be used for the realization of arbitrary voltage transfer functions has been given by Brugler [3]. In this correspondance, it is shown that the above mentioned network can also be used to realize arbitrary transfer admittance functions. Furthermore, it is shown that the generalization of Brugler's circuit results in an active-*RC* network which, with the traditional simple functional decompositions, can be used to realize an arbitrary function as any of the five two-port parameters mentioned above.

### Realization

The circuit shown in Fig. 1 is essentially the network proposed by Brugler [3], where the conductance $G$ is arbitrary. The voltage transfer function is given by

$$\frac{V_2}{V_1} = \frac{Y_1 - Y_2}{Y_3 - Y_4} \tag{1}$$

provided that

$$Y_1 + Y_4 + Y_5 = Y_2 + Y_3 + Y_6. \tag{2}$$

The use of this network to realize an arbitrary ratio of rational polynomials in the $s$-domain as a voltage transfer function is well known. The transfer admittance for this circuit is

$$\frac{I_2}{V_1} = G\,\frac{Y_1 - Y_2}{Y_3 - Y_4} \tag{3}$$

so that the synthesizing arbitrary transfer admittances is essentially the same as that for the case of voltage transfer functions.

However, generalizing this network (as shown in Fig. 2 where $G$ is the arbitrary conductance) results in a circuit whose voltage transfer function is again given by (1). However, the input admittance is

$$\frac{I_1}{V_1} = G\,\frac{Y_2 + Y_3 - Y_1 - Y_4}{Y_3 - Y_4}. \tag{4}$$

The decomposition of an arbitrary ratio of rational polynomials in the $s$-domain into a form such as this is well known [4]–[6]. Thus the network in Fig. 2 can be used to realize arbitrary driving-point admittances. Furthermore, the other two-port parameters for this circuit are as follows.

Current transfer function:

$$\frac{I_2}{I_1} = -\frac{Y_1 - Y_2}{Y_1 + Y_4 - Y_2 - Y_3} \tag{5}$$

Transfer admittance:

$$\frac{I_2}{V_1} = G\,\frac{Y_1 - Y_2}{Y_3 - Y_4} \tag{6}$$

Transfer impedance:

$$\frac{V_2}{I_1} = -\frac{Y_1 - Y_2}{G(Y_1 + Y_4 - Y_2 - Y_3)}. \tag{7}$$

Although the analytical expressions for two-port functions of the network in Fig. 2 are similar to the expressions obtained from the network which has been proposed by Bobrow [2], the major advantage of this alternative approach is that it uses less number of active elements.

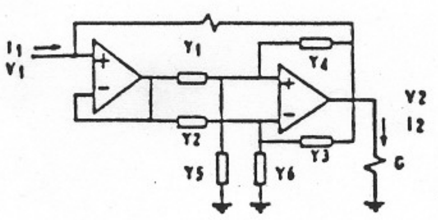


Fig. 2.

Since the proposed technique removes all the restrictions on the type of the realizable two-port functions, this circuit may find a variety of applications where the realizability of a certain type of two-port function is of interest, rather than the sensitivity of the subsystem. In these cases, the proposed circuit is desirable from the viewpoint of less restrictions imposed on the type of functions to be realized.

## Conclusion

The new network allows the realization of arbitrary functions as driving-point admittance, transfer impedance, transfer admittance, and current transfer function, as well as voltage transfer function, by a direct and simple technique. Because the proposed network can be used to realize any one of the five two-port parameters mentioned above, it can be called a universal active-*RC* network. Compared to a recently proposed network [2] employing three operational amplifiers, the new network offers a reduction in the number of necessary active elements for a similar realization. A reduction in the number of passive elements can also be obtained using the rules described in [7] for the realization of high-order two-port parameters.

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# On the Determination of Device Noise and Gain Parameters

MARIO SANNINO

*Abstract*—The least-squares fitting of measured noise figures and gains versus input termination admittance is an established procedure to determine linear two-port noise and gain parameters. Unfortunately, the method is liable to the serious inconvenience of yielding often erroneous results or even results without physical meaning. Some criteria are suggested which allow the carrying out of measurements in such a manner as to safely avoid the above drawbacks.

It is known that the noise figure of a linear two port can be related to the input termination (source) admittance $Y_s = G_s + jB_s$ through the relationship

$$F(Y_s) = F_0 + R_n \frac{|Y_s - Y_0|^2}{G_s} \tag{1}$$

where the minimum noise figure $F_0$, the optimum source admittance $Y_0 = G_0 + jB_0$, and the noise resistance $R_n$ are the two-port noise parameters.

In 1969, Lane presented a computer-aided analytical method to determine the above parameters [1], which offers many advantages from both the accuracy and the experimental convenience viewpoints as compared with the graphic procedures suggested earlier by the IRE Subcommittee on Noise [2].

Lane's method is an application of the least-squares technique which reduces the derivation of the noise parameters to the solution of a four linear equation system obtained as fit of some (redundant) measured noise figures versus source admittances chosen without stringent rules. The procedure is also used to derive gain parameters, and recently, it has been shown in describing a fully automated computer-controlled noise and gain measuring system [3].

Unfortunately, the method, in spite of its advantages, has not always been received with favor by experimenters, because it is liable to the serious inconvenience of yielding often erroneous results or, in some cases, even results without physical meaning. On the other hand, this was noted by the author himself, who consequently concluded his paper by inviting suggestions for a more sophisticated data processing technique with lower error sensitivity.

Actually, under some experimental conditions, i.e., depending on the admittances chosen to perform measurements, the ill-conditioning of the coefficient matrix of the equation system to solve may occur, with a consequent large increase in the error sensitivity. In this case, the only way to obtain correct results is to repeat all the measurements for different values of the source admittance.

Allowing for the above considerations, the present note may prove useful in that:

1) it shows that the above mentioned ill-conditioning situation occurs when the chosen admittances do lie in the neighborhoods of some *singular loci* on the $G_s$, $B_s$ plane;
2) it proposes some measurement criteria to follow in order to surely avoid such drawbacks, together with suggestions for practical application of these criteria.

To prove this, let us consider the symmetric coefficient-matrix of the four linear-equations system which is to be solved in accordance with the least-squares method as applied in [1]. It is given by

$$\begin{bmatrix} n & & & (a_{jk} = a_{kj}) \\ \sum_{i=1}^{n} \left(G_{si} + \frac{B_{si}^2}{G_{si}}\right) & \sum_{i=1}^{n} \left(G_{si} + \frac{B_{si}^2}{G_{si}}\right)^2 & & \\ \sum_{i=1}^{n} \frac{1}{G_{si}} & \sum_{i=1}^{n} \left(1 + \frac{B_{si}^2}{G_{si}^2}\right) & \sum_{i=1}^{n} \frac{1}{G_{si}^2} & \\ \sum_{i=1}^{n} \frac{B_{si}}{G_{si}} & \sum_{i=1}^{n} B_{si} + \frac{B_{si}^3}{G_{si}^2} & \sum_{i=1}^{n} \frac{B_{si}}{G_{si}^2} & \sum_{i=1}^{n} \left(\frac{B_{si}}{G_{si}}\right)^2 \end{bmatrix} \tag{2}$$

where $n$ is the number of the sets of experimental data $F_i$, $G_{si}$, $B_{si}$ processed (with $i = 1 \div n$).

By inspection of this matrix, some values of the input termination admittance $Y_s$ can be recognized which can cause the matrix to become singular, in which case, solutions do not exist because the surface (1) is not completely defined. In general, singularity of the matrix occurs for every locus of $Y_s$ which represents a projection on the plane $G_s$, $B_s$ (or on the Smith chart) of a curve which can be thought of as an intersection of the paraboloid $F(Y_s)$ given by (1) with other ones of the same family.

Some of these values are the ones for which a column (row) is proportional to another one or can be obtained as a linear combination of other ones. Therefore, loci of $Y_s$ which cause singularity of the matrix

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